governor may also be cited. The whirligig may be considered a practical reversion of the principle exemplified in this problem.

AVERAGE AND PROBABILITY.

NOTE ON THE POND PROBLEM.

It was our purpose to publish a second solution of problem 90, Average and Probability, by a different method, but so far we have been unable to obtain a positive result. We have gone over our investigations a number of times, and have had several of our contributors go over the calculations, and so far we have been unable to find our error. Since we obtain a negative result, there is certainly something wrong with the work, for the problem admits of a definite solution when once the law of distribution has been decided upon. We hope to find time to go over our work again, and should we find our error, we will publish our solution as the method may be of interest to many of our readers.

This problem was originally proposed by Artemas Martin, Ph. D., and published as problem 300, Vol. I., No. 6, page 195, in the *Mathematical Visitor*, edited and published by himself. We infer that this is the source from which Dr. Byerly took it for his problem 21, *Integral Calculus*, second edition, page 211. While no specific reference is made as to the source from which it was taken, yet in his introductory paragraph on the subject of Mean Value and Probability, Dr. Byerly makes mention of the *Mathematical Visitor*.

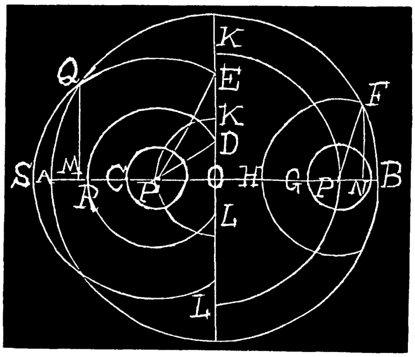
In a letter to us from Dr. Martin, he says that Professor Seitz sent him a solution, giving the same answer as was obtained in the two solutions published in the December number of Vol. VII. ED. F.

106. **Proposed by L. C. WALKER, A. M., Petaluma High School, Petaluma, Cal.**

Required the average distance between two points in a hemisphere.

Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

Let O be the center of the sphere, P any point in the radius AO or BO. Take the left hemisphere when $OP<\frac{1}{2}r$, the right hemisphere when $OP>\frac{1}{2}r$. Let $OP=x$, PC, PG, PD, PE, $PF=y$.



If the first point is anywhere on the hemisphere with radius x, the second point must be on the surface of the zone of a sphere radius y. The surface of the hemisphere radius x is $2\pi x^2$ when $x<\frac{1}{2}r$. If $y<x$ the second point lies on the surface of a sphere radius y. The area of this surface$=4\pi y^2$. If $y>x$ and $<(r-x)$, the second point is on the area $2\pi PR.OR=$

$2\pi y(y+x)$. If $y>(r-x)$ and $<\sqrt{(r^2+x^2)}$, the second point lies on the area $2\pi OS.OM=(\pi y/x)(r^2+x^2-y^2)$. When $x>\frac{1}{2}r$. The only different area for the second point is when $y>(r-x)$ and $<x$; then the area is $2\pi PH.NH=(\pi y/x)[r^2-(y-x)^2]$. Let $\triangle$ be the required average distance.

$$\triangle=\frac{2\pi^2}{(\frac{4}{3}\pi r^3)^2}\int_0^{\frac{1}{2}r} x^2dx\left[\int_0^x 4y^3dy+\int_x^{r-x} 2y^2(y+x)dy+\int_{r-x}^{\sqrt{(r^2+x^2)}}(y^2/x)(r^2+x^2-y^2)dy\right]$$

$$+\frac{2\pi^2}{(\frac{4}{3}\pi r^3)^2}\int_{\frac{1}{2}r}^r x^2dx\left[\int_0^{r-x}4y^3dy+\int_{r-x}^x(y^2/x)[r^2-(y-x)^2]dy\right.$$

$$\left.+\int_x^{\sqrt{(r^2+x^2)}}(y^2/x)(r^2+x^2-y^2)dy\right]$$

$$=\frac{3}{20r^6}\int_0^r [15r^4x^2-20r^3x^3+10r^2x^4-6x^6-4r^5x+4x(r^2+x^2)^{\frac{5}{2}}]dx$$

$$=\tfrac{3}{70}(16\sqrt{2}-5)r.$$

If one point is taken in each hemisphere we get

$$\triangle_1=\frac{2\pi^2}{(\frac{4}{3}\pi r^3)^2}\int_0^r x^2dx\left[\int_x^{\sqrt{(r^2+x^2)}}2y^2(y-x)dy+\int_{\sqrt{(r^2+x^2)}}^{r+x}(y^2/x)[r^2-(y-x)^2]dy\right]$$

$$=\frac{3}{20r^6}\int_0^r[4r^5x+15r^4x^2+20r^3x^3+10r^2x^4+4x^6-4x(r^2+x^2)^{\frac{5}{2}}]dx$$

$$=\tfrac{3}{70}(53-16\sqrt{2})r.$$

If both points are taken anywhere in the sphere we get

$$\triangle_2=\frac{4\pi^2}{(\frac{4}{3}\pi r^3)^2}\int_0^r x^2dx\left[\int_0^{r-x}4y^3dy+\int_{r-x}^{r+x}(y^2/x)[r^2-(y-x)^2]dy\right]$$

$$=\frac{3}{20r^6}\int_0^r(15r^4+10r^2x^2-x^4)x^2dx=\frac{36r}{35}.$$

$$\triangle=2\triangle_2-\triangle_1=\tfrac{3}{70}(16\sqrt{2}-5)r.$$

These methods greatly simplify the tedium of integration.

MISCELLANEOUS.

---

98. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Chemistry and Physics, The Temple College, Philadelphia, Pa.

A particle describes an ellipse under an attraction always directed to the vertex; to determine the law of the attraction.